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COORDINATE TRANSFORMATIONS USED IN OGO SATELLITE DATA ANALYSIS

JANUARY 1970

GSFC ── **GODDARD SPACE FLIGHT CENTER** ──
GREENBELT, MARYLAND

COORDINATE TRANSFORMATIONS USED

IN OGO SATELLITE DATA ANALYSIS

MAGNETIC AND ELECTRIC FIELDS BRANCH

JANUARY 1970

Goddard Space Flight Center
Greenbelt, Maryland

COORDINATE TRANSFORMATIONS USED
IN OGO SATELLITE DATA ANALYSIS

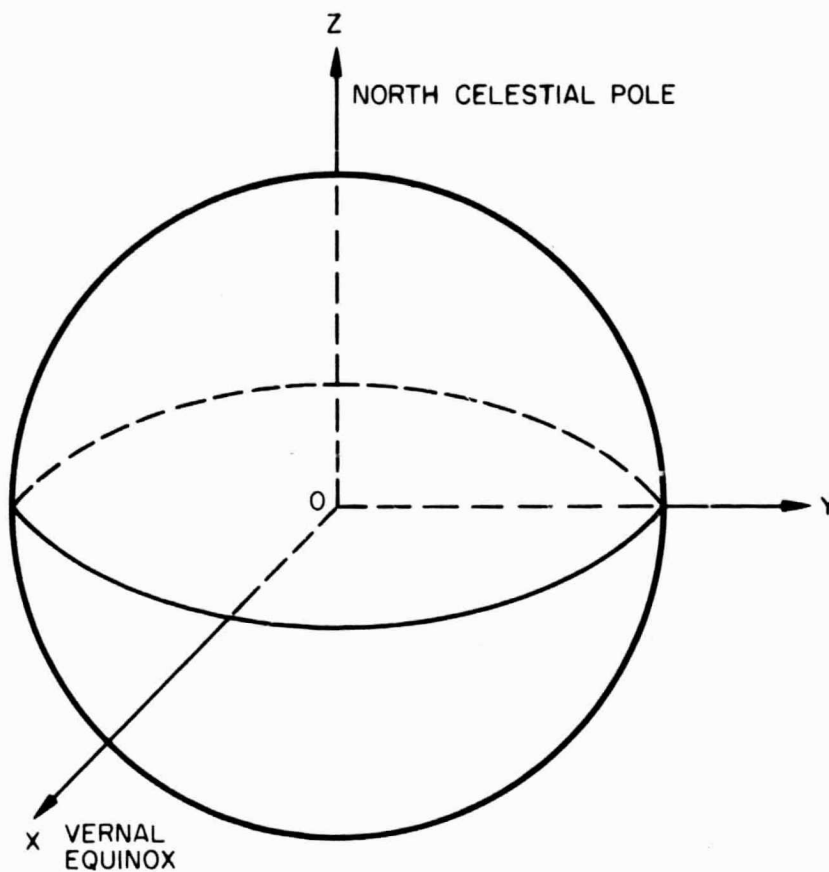## 1. Geocentric Celestial Inertial Coordinates: GCI System

Figure 1. Geocentric celestial inertial coordinates (gci system). The Z axis points at the north celestial pole; the X axis points at the vernal equinox; and the Y axis completes a right-handed orthogonal system. The XY plane coincides with the earth's equatorial plane.

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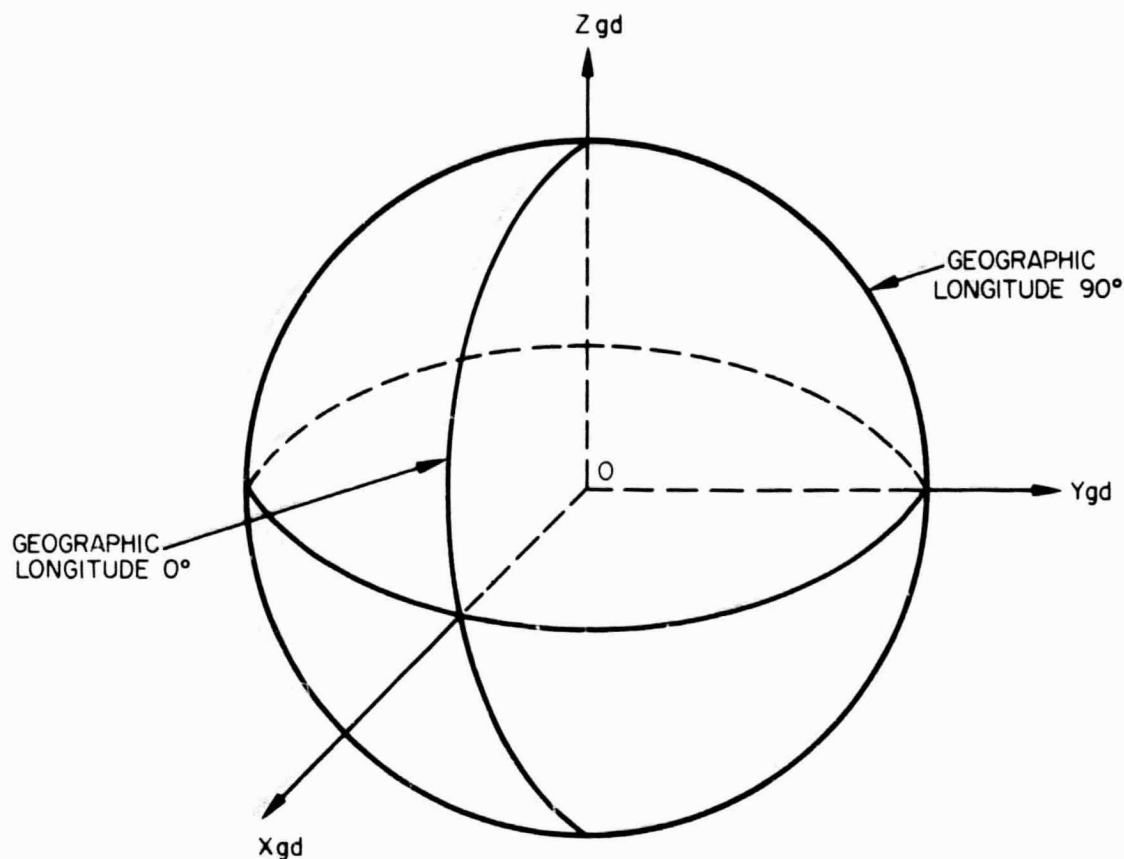## 2. Geographic (Distance) Coordinates: GD System

Figure 2. Geographic (distance) coordinates (gd system). The Z_{gd} axis points at the north celestial pole. The $X_{gd}Y_{gd}$ plane coincides with the equatorial plane, the X_{gd} and Y_{gd} axes being intersections of the meridian planes at geographic longitude $0°$ and $90°$ E with the equatorial plane, respectively.

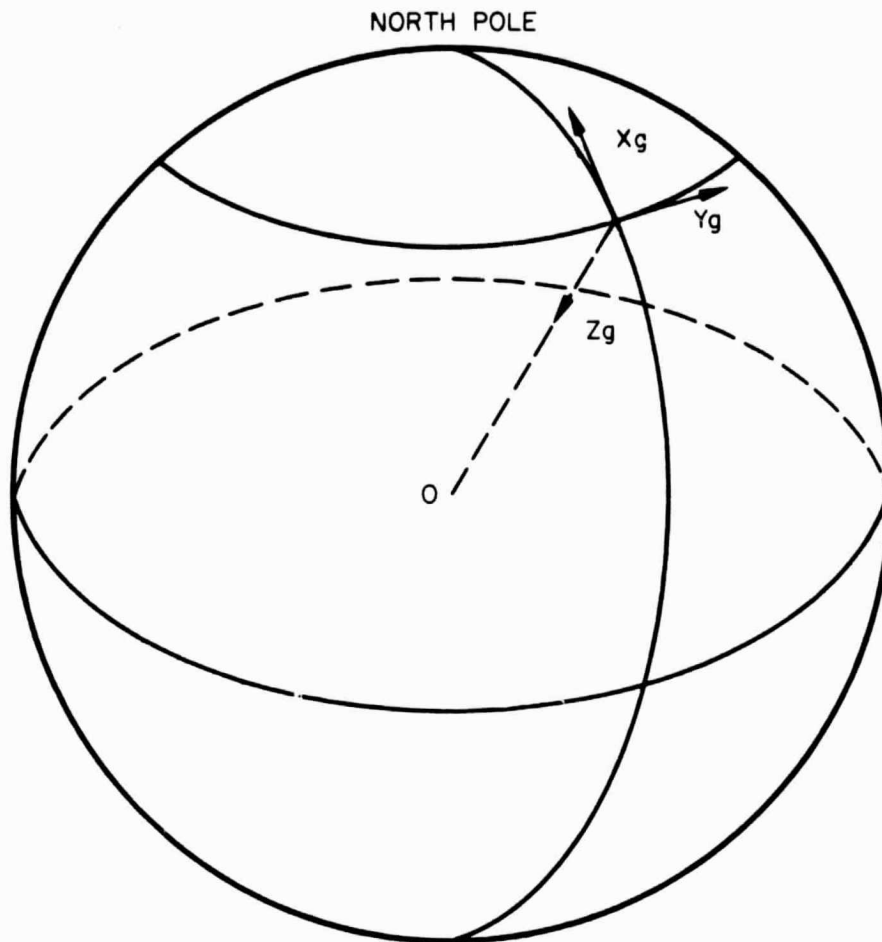## 3. Topographic (Geographic Direction) Coordinates: G System

Figure 3. Topographic (geographic direction) coordinates (g system). On any spherical surface concentric with the earth the X_g axis points to the north, the Y_g axis to the east, and the Z_g axis to the center of the earth.
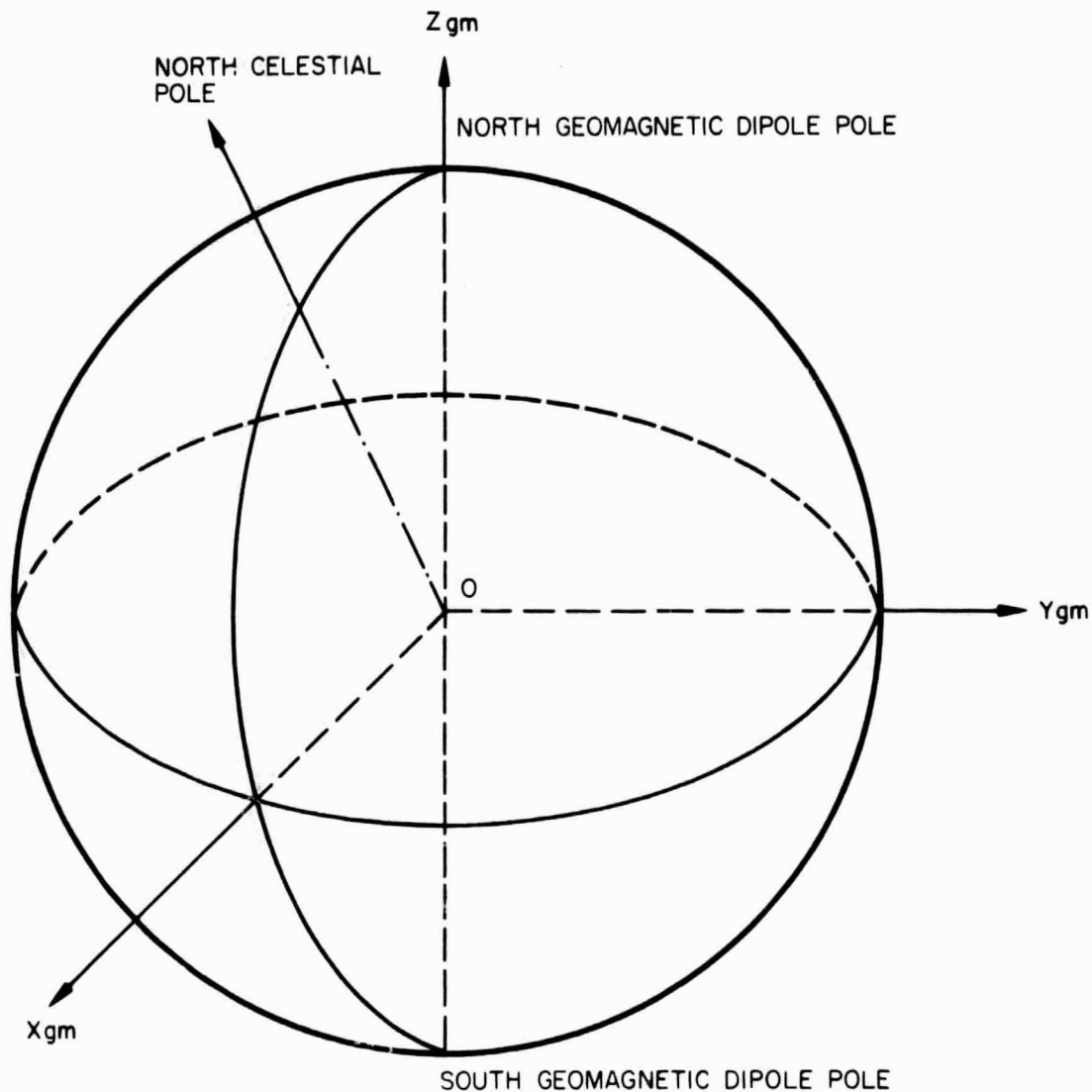
4. Geomagnetic Coordinates: GM System

Figure 4. Geomagnetic coordinates (gm system). The Z_{gm} axis points at the north geomagnetic dipole pole. The X_{gm} axis is the intersection of the geomagnetic dipole equatorial plane and the geomagnetic dipole meridian plane containing the north celestial pole. The Y_{gm} axis completes a right-handed orthogonal system. The geographic co-latitude, θ_{gm}, and the geographic east longitude, ϕ_{gm}, of the north geomagnetic dipole pole are taken to be: $\theta_{gm} = 11.4°$, and $\phi_{gm} = 290°$ $(=-70°)$.

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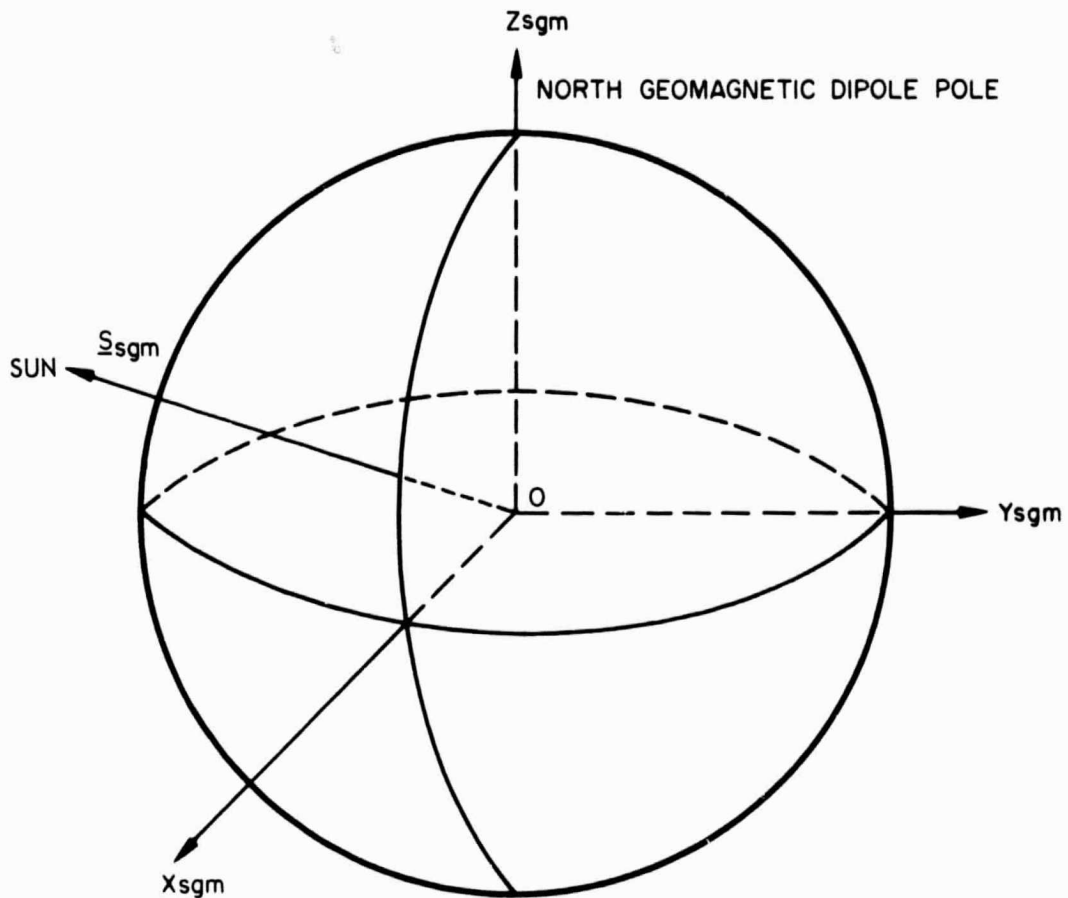## 5. Solar Geomagnetic Coordinates: SGM System

Figure 5. Solar geomagnetic coordinates (sgm system). The Z_{sgm} axis points to the north geomagnetic dipole pole. The X_{sgm} axis is defined by the intersection of the geomagnetic dipole equatorial plane and the geomagnetic dipole meridian plane containing the sun. The Y_{sgm} axis completes a right-handed orthogonal system. The solar vector, S_{sgm}, lies in the $X_{sgm} Z_{sgm}$ plane.

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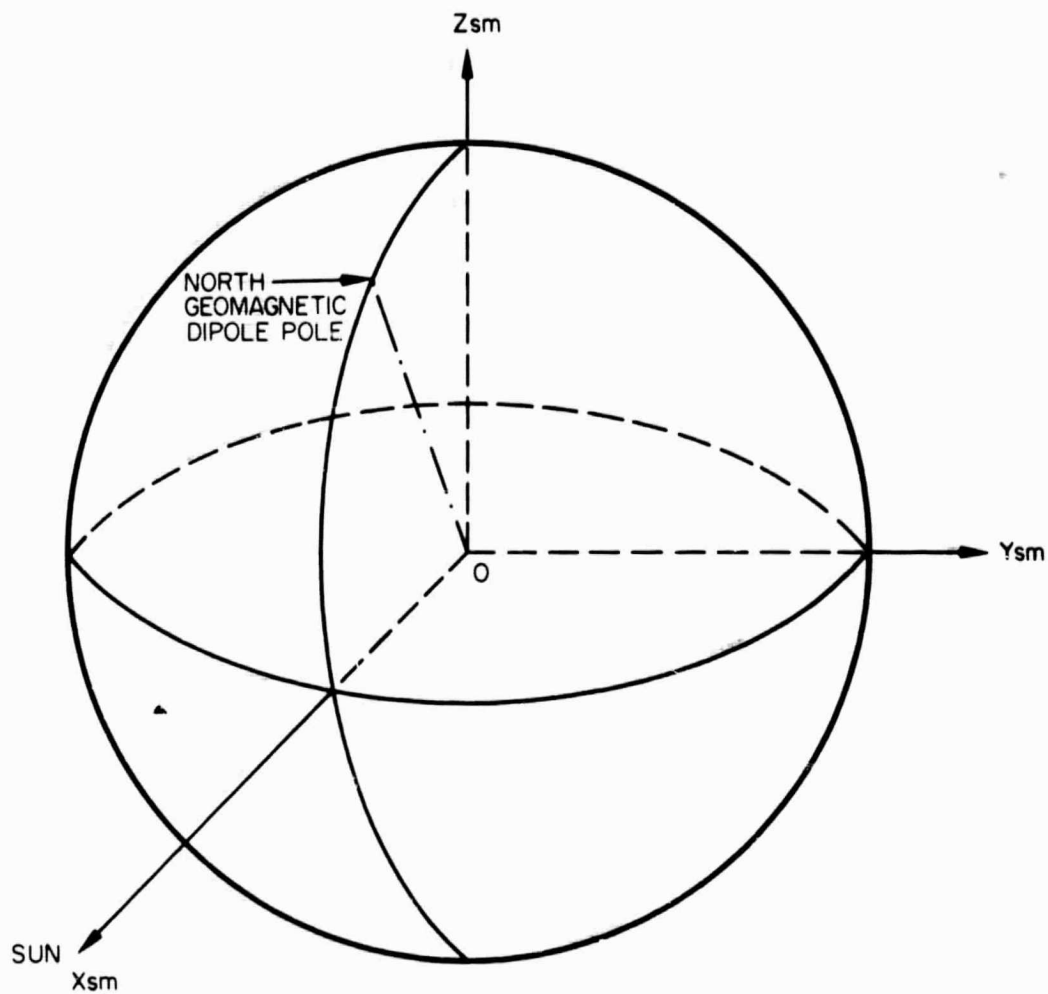## 6.   Solar Magnetospheric Coordinates: SM System

Figure 6. Solar magnetospheric coordinates (sm system). The X_{sm} axis points at the sun. The $X_{sm} Z_{sm}$ plane contains the north geomagnetic dipole pole. The Y_{sm} axis completes a right-handed orthogonal system.

7. Solar Ecliptic Coordinates (SE System)
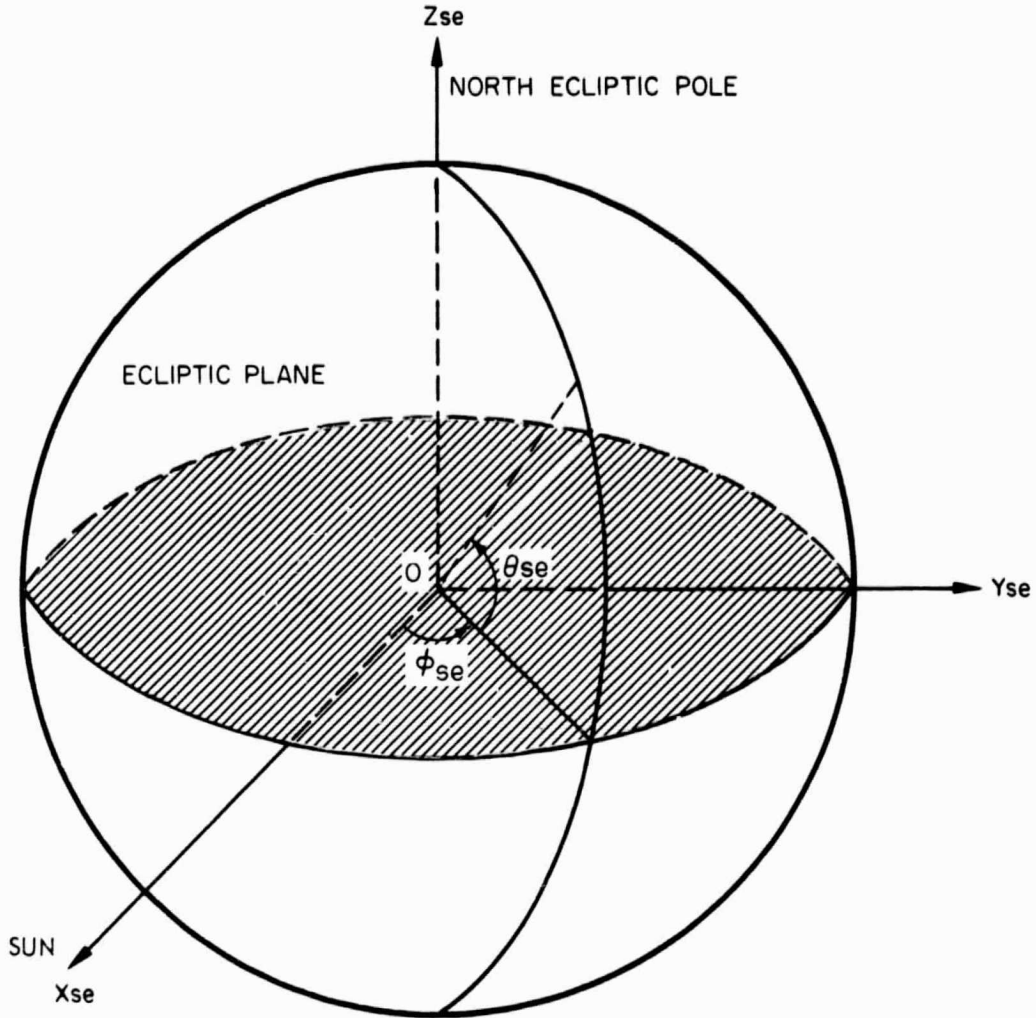
Figure 7. Solar ecliptic coordinates (se system). The X_{se} axis points at the sun. The X_{se} Y_{se} plane coincides with the solar ecliptic plane, and the Z_{se} axis points at the north ecliptic pole. Solar ecliptic latitude, θ_{se}, and solar ecliptic longitude, ϕ_{se}, are given by:

$$\theta_{se} = \sin^{-1}\{Z_{se}/(X_{se}^2 + Y_{se}^2 + Z_{se}^2)^{\frac{1}{2}}\}$$

$$\phi_{se} = \tan^{-1}(Y_{se}/X_{se}) = \cos^{-1}\{X_{se}/(X_{se}^2 + Y_{se}^2)^{\frac{1}{2}}\}$$

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8. Coordinate Transformations From GCI Coordinates to Other Coordinates

8.1 Symbols

α = satellite right ascension

δ = satellite declination (= satellite geographic latitude)

λ = satellite geographic longitude

ϵ = 23° 27' 8.26" = obliquity of the ecliptic plane

θ_{gm} = 11.4° = geographic co-latitude of the north geomagnetic dipole pole

ϕ_{gm} = 290° (= –70°) = geographic longitude of the north geomagnetic dipole pole

β = geomagnetic longitude of the sun

ψ = geomagnetic longitude of the satellite

γ = geomagnetic latitude of the sun

S = solar vector in g.c.i. system (given in km)

8.2 Transformation Matrices

(a) $\underline{\underline{G}}$ matrix: gci system to g system

$$\underline{\underline{G}} = \begin{pmatrix} -\sin\delta\cos\alpha & -\sin\delta\sin\alpha & \cos\delta \\ -\sin\alpha & \cos\alpha & 0 \\ -\cos\delta\cos\alpha & -\cos\delta\sin\alpha & -\sin\delta \end{pmatrix}$$

(b) $\underline{\underline{J}}$ matrix: gci system to gd system

$$\underline{\underline{J}} = \begin{pmatrix} \cos(\alpha-\lambda) & \sin(\alpha-\lambda) & 0 \\ -\sin(\alpha-\lambda) & \cos(\alpha-\lambda) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c) $\underline{\underline{R}}$ matrix: gd system to gm system

$$\underline{\underline{R}} = \begin{pmatrix} \cos\theta_{gm} \cos\phi_{gm} & \cos\theta_{gm} \sin\phi_{gm} & -\sin\theta_{gm} \\ -\sin\phi_{gm} & \cos\phi_{gm} & 0 \\ \sin\theta_{gm} \cos\phi_{gm} & \sin\theta_{gm} \sin\phi_{gm} & \cos\theta_{gm} \end{pmatrix}$$

(d) $\underline{\underline{V}}$ matrix: gm system to sgm system

$$\underline{\underline{V}} = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

β (= geomagnetic longitude of the sun) can be determined as follows: The solar vector, S_{gm} , in geomagnetic coordinates is obtained from the solar vector, S, in gci coordinates by

$$S_{gm} = \underline{\underline{R}} \, \underline{\underline{J}} \, S$$

Writing components of S_{gm} as ($s_{gm,1}$, $s_{gm,2}$, $s_{gm,3}$), β is given by

$$\beta = \tan^{-1} (s_{gm,2} / s_{gm,1}) = \cos^{-1} \{ s_{gm,1} / (s^2_{gm,1} + s^2_{gm,2})^{1/2} \}$$

(e) $\underline{\underline{W}}$ matrix: sgm system to sm system

$$\underline{\underline{W}} = \begin{pmatrix} \cos\gamma & 0 & \sin\gamma \\ 0 & 1 & 0 \\ -\sin\gamma & 0 & \cos\gamma \end{pmatrix}$$

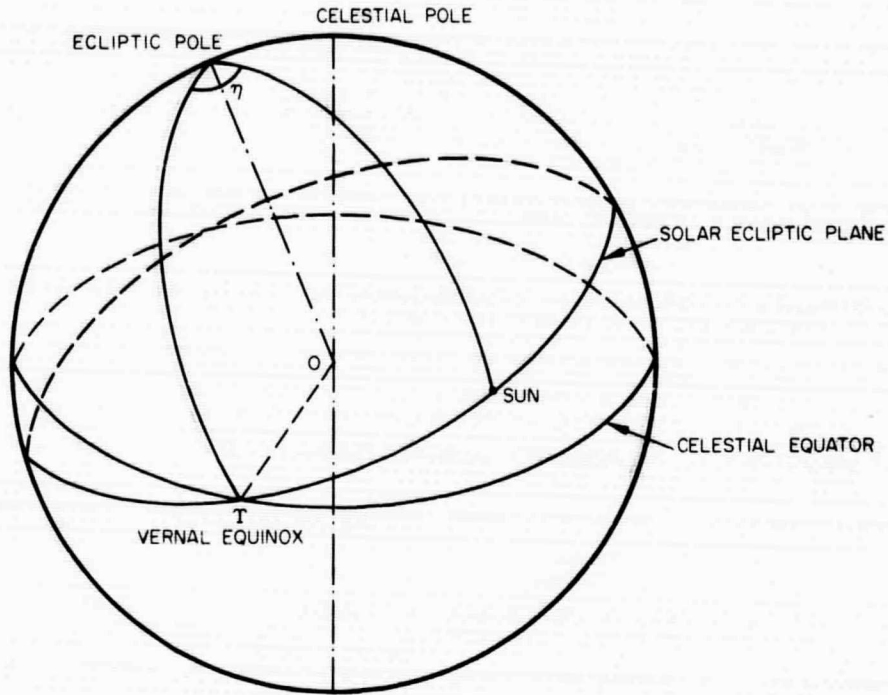γ (= geomagnetic latitude of the sun) can be determined by

Figure 8. Angle η

$$\gamma = \tan^{-1} (s_{gm,3} / s_{gm,1})$$

See (d) for the components of the sun vector, S_{gm}, in geomagnetic coordinates.

(f) $\underline{\underline{H}}$ matrix: gci system to so system

$$\underline{\underline{H}} = \begin{pmatrix} \cos \eta & \sin \eta \cos \epsilon & \sin \eta \sin \epsilon \\ -\sin \eta & \cos \eta \cos \epsilon & \cos \eta \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

where η is the angle between the X_{gci} axis (= the vernal equinox) and the sun vector (Fig. 8), namely,

$$\eta = \cos^{-1} \{ s_1 / (s_1^2 + s_2^2 + s_3^2)^{1/2} \}$$

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Here s_1, s_2, and s_3 are the X_{gci}, Y_{gci}, and Z_{gci} components of the sun vector, S, respectively.

8.3   Transformations of a Vector A in the gci System

(a) To the topographic (g) system:

$$A_g = \underline{\underline{G}}\, A$$

(b) To the geographic distance (gd) system:

$$A_{gd} = \underline{\underline{J}}\, A$$

(c) To the geomagnetic (gm) system:

$$A_{gm} = \underline{\underline{R}}\, A_{gd} = \underline{\underline{R}}\,\underline{\underline{J}}\, A$$

(d) To the solar geomagnetic (sgm) system:

$$A_{sgm} = \underline{\underline{V}}\, A_{gm} = \underline{\underline{V}}\,\underline{\underline{R}}\,\underline{\underline{J}}\, A$$

(e) To the solar magnetospheric (sm) system:

$$A_{sm} = \underline{\underline{W}}\, A_{sgm} = \underline{\underline{W}}\,\underline{\underline{V}}\,\underline{\underline{R}}\,\underline{\underline{J}}\, A$$

(f) To the solar ecliptic (se) system:

$$A_{se} = \underline{\underline{H}}\, A$$

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9. Expressing a Vector **A** by the Magnitude and Two Angles with Respect to the Solar Ecliptic Coordinate System.

Given a vector **A** at a point P in space, the vector can be described by its magnitude $|\mathbf{A}|$ and two angles θ_{se} and ϕ_{se}, where θ_{se} and ϕ_{se} are respectively solar ecliptic latitude and longitude given by

$$\theta_{se} = \tan^{-1}\left\{A_{se,3} / (A_{se,1}^2 + A_{se,2}^2)^{1/2}\right\}$$

$$= \sin^{-1}(A_{se,3}/A)$$

$$\phi_{se} = \tan^{-1}(A_{se,2}/A_{se,1})$$

$$= \cos^{-1}\left\{A_{se,1} / (A_{se,1}^2 + A_{se,2}^2)^{1/2}\right\}$$

where $A_{se,1}$, $A_{se,2}$, and $A_{se,3}$ are X_{se}, Y_{se}, and Z_{se} components of **A** and $A = |\mathbf{A}|$.

10. Expressing a Vector **A** by the Magnitude and Two Angles, D and I .

Given a vector **A** at a point P in space, the vector can be described by its magnitude $|\mathbf{A}|$, and two angles, declination D and inclination I.

Let $A_{g,1}$, $A_{g,2}$, and $A_{g,3}$ be X_g, Y_g, and Z_g components of **A**. Then D and I are given by

$$D = \cos^{-1}\left\{A_{g,1} / (A_{g,1}^2 + A_{g,2}^2)^{1/2}\right\}$$

$$= \tan^{-1}(A_{g,2}/A_{g,1})$$

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$$I = \sin^{-1}(A_{g,3}/A)$$

$$= \tan^{-1}\{A_{g,3}/(A_{g,1}^2 + A_{g,2}^2)^{1/2}\}$$

where

$$A = |\mathbf{A}|$$